647510 **BSCgraCC6010** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
March/April-2024 (OLD COURSE)
GRAPH THEORY AND COMPLEX ANALYSIS -2(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) ATTEMPT ANY ONE **(07)**

(1) State and prove the first theorem of Graph Theory.

(2) Define: Connected Graph
Prove that a graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ is disconnected ***iff*** its vertex set $\boldsymbol{V}$ can be partitioned into two nonempty, disjoint subsets $\boldsymbol{V_1}$ & $\boldsymbol{V_2}$ such that there exists no edge in $\boldsymbol{G}$ whose one end vertex is in subset $\boldsymbol{V_1}$ and the other in subset $\boldsymbol{V_2}$.

Que-1(B) ATTEMPT ANY ONE **(04)**

(1) Define: (i) Vertex-disjoint subgraph (ii) Edge-disjoint subgraph (iii) Closed Walk (iv) Euler graph.

(2) A simple graph with $\boldsymbol{n}$ vertices and $\boldsymbol{k}$ components can have at most $\frac{(\boldsymbol{n}-\boldsymbol{k})(\boldsymbol{n}-\boldsymbol{k}+\mathbf{1})}{\mathbf{2}}$ edges.

Que-1(C) ATTEMPT ANY ONE **(03)**

(1) Define: (i) Degree of a vertex (ii) Pendant vertex (iii) Isolated vertex

(2) Show that an infinite graph with a finite number of vertices will have at least one pair of vertices joined by an infinite number of parallel edges.

Que-2(A) ATTEMPT ANY ONE **(07)**

(1) Define: Embedding , Planar Graph
Prove that the Kuratowski's first graph is non-planar.

(2) Prove that the rank of the incidence matrix of a connected graph $\boldsymbol{G}$ with $\boldsymbol{n}$ vertices is $\boldsymbol{n} - \mathbf{1}$.

Que-2(B) ATTEMPT ANY ONE **(04)**

(1) Prove that the vertex connectivity of a graph can never exceed its edge connectivity.

(2) State the Euler's Formula for planar graphs.
If $\boldsymbol{G}$ be a graph with $\boldsymbol{n}$ vertices, $\boldsymbol{e}$ edges and $\boldsymbol{f}$ faces, then prove that
(i) $\boldsymbol{e} \geq \frac{3}{2}\boldsymbol{f}$ (ii) $\boldsymbol{e} \leq \mathbf{3}\boldsymbol{n} - \mathbf{6}$.

Que-2(C) ATTEMPT ANY ONE **(03)**

(1) Define: Adjacency Matrix.
Write any three observations of the adjacency matrix of a graph.

(2) Define: (i) Proper Coloring (ii) Chromatic Number (iii) Independence Number

Que-3(A) ATTEMPT ANY ONE **(07)**

(1) Explain: Conformal Mapping.

(2) Discuss the transformation $\boldsymbol{w} = \boldsymbol{cosz}$.

Que-3(B) ATTEMPT ANY ONE **(04)**

(1) Prove that the composition of two bilinear maps is a bilinear map.

(2) Show that Bilinear mapping $\boldsymbol{w} = \frac{\boldsymbol{az}+\boldsymbol{b}}{\boldsymbol{cz}+\boldsymbol{d}}$, $\boldsymbol{z} \neq \frac{-\boldsymbol{d}}{\boldsymbol{c}}$, $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ & $\boldsymbol{d} \in \mathbb{C}$, $\boldsymbol{ad} - \boldsymbol{bc} \neq \mathbf{0}$ is conformal.

Que-3(C) ATTEMPT ANY ONE **(03)**

(1) Find the critical points of the Mobious mapping.

(2) Discuss the transformation $\boldsymbol{w = e^z}$ in polar form.

Que-4(A) ATTEMPT ANY ONE **(07)**

(1) Define: Region & Radius of Convergence of a power series.

Find region and radius of convergence for

(i) $\sum_{n=1}^{\infty} \frac{z^n}{7^n+1}$ and (ii) $\sum_{n=1}^{\infty} \frac{z^n}{9^n-1}$.

(2) Define: Analytic Function

State and prove Taylor's infinite series of an analytic function $\boldsymbol{f(z)}$.

Que-4(B) ATTEMPT ANY ONE **(04)**

(1) Show that $\boldsymbol{\frac{sinhz}{z^2} = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{z^{2n-1}}{(2n+1)!}}$.

(2) Write Laurent'z series of $\boldsymbol{\frac{e^z}{z(1+z^2)}}$ for $\boldsymbol{|z| < 1}$.

Que-4(C) ATTEMPT ANY ONE **(03)**

(1) Define: (i) Complex sequence (ii) Power series

(iii) Absolutely convergent series.

(2) Expand $\boldsymbol{\log(1+z)}$ in the powers of $\boldsymbol{z}$.

Que-5(A) ATTEMPT ANY ONE **(07)**

(1) Define: Pole.

If $\boldsymbol{z_0}$ is the $\boldsymbol{m^{th}(m \geq 2)}$ order pole of a complex function $\boldsymbol{f(z)}$, then prove that

$\boldsymbol{Res(f(z), z_0) = \frac{\varphi^{m-1}(z_0)}{(m-1)!}}$, where $\boldsymbol{\varphi(z) = (z - z_0)^m f(z)}$.

(2) State the Cauchy's residue theorem and evaluate $\int_C \frac{e^z}{z(z-1)^2} dz$, $C: |z| = 2$.

Que-5(B) ATTEMPT ANY ONE **(04)**

(1) Evaluate: $\boldsymbol{\int_0^{\infty} \frac{1}{(x^2+1)^2} dx}$.

(2) Evaluate: $\boldsymbol{\int_0^{\pi} \frac{d\theta}{(2+cos\theta)^2}}$.

Que-5(C) ATTEMPT ANY ONE **(03)**

(1) Evaluate: $\int_0^{2\pi} \frac{d\theta}{\frac{5}{4}+sin\theta}$.

(2) Evaluate: $\int_0^{\infty} \frac{x^2}{(x^2+4)(x^2+9)} dx$.

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